AP Calculus AB
Test: Inverse and Transcendental Functions
Instructor's Version

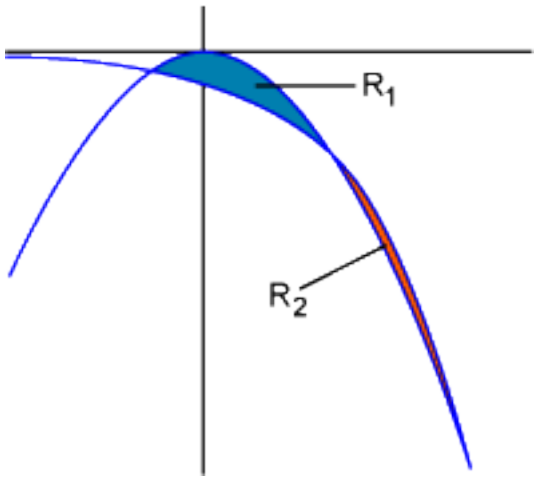
Free-Response Question 1 (with calculator)
You may use your calculator for this question.
(When you finish this question, if your test is proctored, ask your proctor for the next section. If you're taking the unproctored version of the test, continue with the next question.)

1. The shaded regions R_1 and R_2, shown below, are enclosed by the graphs of $f(x)=-x^2$ and $g(x)=-2^x$.

A. Find the *x*- and *y*-coordinates of the three points of intersection of the graphs of *f* and *g*.

Answer: $(-0.767, -0.588)$
$(2, -4)$ and $(4, -16)$

(2 points)



B. Estimate the area of R_1 to three reasonable places past the decimal.

Answer: $\int_{-0.7666647}^{2} \left(-x^2 + 2^x\right) dx \approx 2.106$

1 point for integrand.
1 point for limits of integration.
1 point answer.

Note: A decimal answer within ± 0.001 is acceptable. Points may be awarded if student works part B correctly using an incorrect value from part A.

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C. Find the EXACT area of R_2. (No calculator approximations; don't simplify.)

Answer:
$$A=\int_2^4\left(-2^x+x^2\right)dx=\left[-\frac{1}{\ln 2}2^x+\frac{1}{3}x^3\right]_2^4=\left(-\frac{1}{\ln 2}2^4+\frac{1}{3}4^3\right)-\left(-\frac{1}{\ln 2}2^2+\frac{1}{3}2^3\right)$$
$$=\frac{64}{3}-\frac{16}{\ln 2}-\frac{8}{3}+\frac{4}{\ln 2}=\frac{56}{3}-\frac{12}{\ln 2}$$

1 point for integrand
1 point for limits of integration
1 point for antiderivative
2 points for answer (no need to simplify)

D. Find the exact volume of the solid formed by revolving R_2 about the x-axis. (No calculator approximations; don't simplify.)

Answer:
$$V=\int_2^4\pi\left(\left(x^2\right)^2-\left(2^x\right)^2\right)dx=\int_2^4\pi\left(\left(x^4\right)-\left(2^{2x}\right)\right)dx=\pi\left[\frac{1}{5}x^5-\frac{1}{2\ln 2}2^{2x}\right]_2^4$$
$$=\pi\left[\frac{1}{5}4^5-\frac{1}{2\ln 2}2^8\right]-\pi\left[\frac{1}{5}2^5-\frac{1}{2\ln 2}2^4\right]=\pi\left(\frac{1024}{5}-\frac{128}{\ln 2}\right)-\pi\left(\frac{32}{5}-\frac{8}{\ln 2}\right)$$
$$=\pi\left(\frac{992}{5}-\frac{120}{\ln 2}\right)$$

1 point for integrand
1 point for constant π and limits of integration
1 point for antiderivative
2 points for answer

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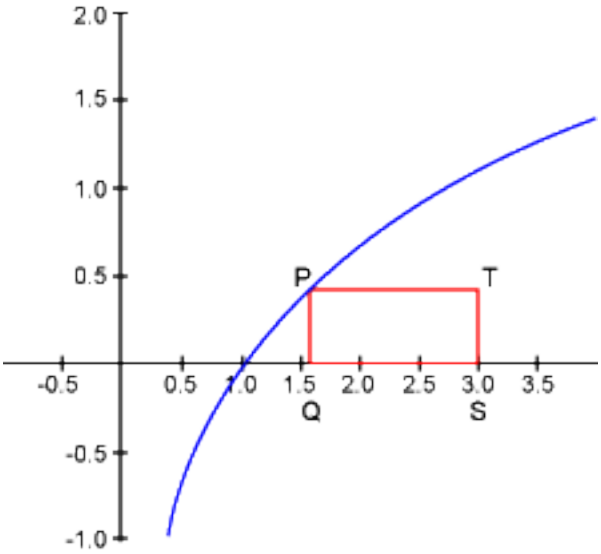
Free-Response Question 2 (no calculator)
You may not use your calculator for this question.

2. The region R is a rectangle with vertices P $(a, \ln a)$, Q $(a, 0)$, S $(3, 0)$, and T $(3, \ln a)$, where $1 < a < 3$.

A. Write an expression that gives the area of the rectangle as a function of *a*.

Answer: Avg = $(\ln a)(3 - a)$

(2 points)



B. The area of the rectangle is maximized for some *c* between 1 and 3. Write the expression you would need to solve in order to find *c*. ***You don't need to find c.***

Answer:

$$A' = \frac{1}{a}(3-a) + -\ln a \qquad \text{(2 points)}$$

$$= \frac{3}{a} - \ln a - 1 = 0 \qquad \text{(2 points)}$$

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C. What is the rate of change of the area when $a=3$, if a is decreasing at a rate of $\frac{da}{dt}=-0.5$ units/sec? You don't need to simplify, of course. Use appropriate units.

$$A'(a)=\frac{1}{a}(3-a)+-\ln a$$

Answer: $\frac{dA}{dt}=\left(\frac{1}{a}(3-a)+-\ln a\right)\frac{da}{dt}$ (1.5 points)

$$\frac{dA}{dt}=\left(\frac{1}{3}(3-3)+-\ln 3\right)(-0.5)\ \frac{\text{units}^2}{\text{sec}}\approx 0.549\ \frac{\text{units}^2}{\text{sec}}$$ (1.5 points)

(Units are worth an additional 1 point.)

Note: If students are correctly using a wrong answer from part A, they should get full credit.

D. Again, a is decreasing at the rate of $\frac{da}{dt}=-0.5$ units/sec. At $a=2.9$, is the area of the rectangle increasing or decreasing? How do you know? Use appropriate units in your answer.

Answer: $\frac{dA}{dt}=\left(\frac{1}{2.9}(3-2.9)+-\ln 2.9\right)(-0.5)\approx 0.5151\ \frac{\text{units}^2}{\text{sec}}$ (2 points)

The area is increasing, because the rate of change is positive. (2 points; units are worth an additional 1 point.)

Note: The students could easily guess that the area is increasing, but they need to do some estimation and calculation to get the reasoning point.

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